Nested Stochastics in Life Insurance

What is “nested stochastics” and why do you need “nested stochastics”
The crucial steps to make nested stochastic simulations feasible
Accurate fast models increase the transparency of the risk exposure
Why “nested stochastics” is the most efficient method for solvency and ALM
Embedding all solvency calculation techniques in a unifying framework

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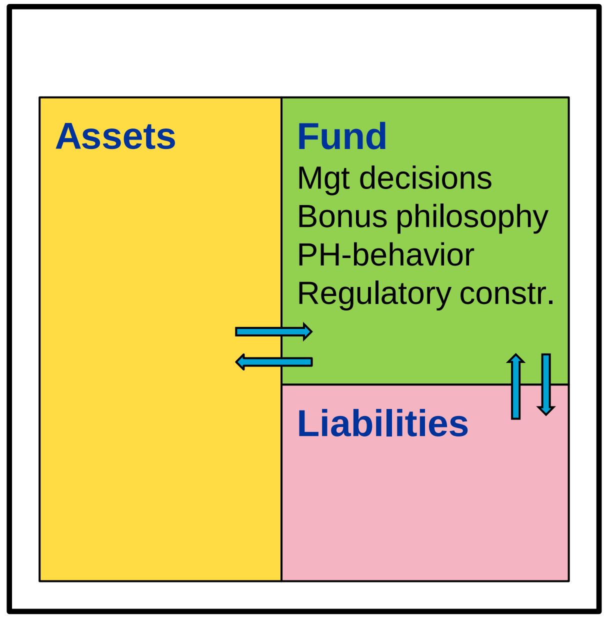
Core of all modern valuation frameworks: MCEV engine

- Economic scenarios
- Financial data / Assets
- Policy data / Liabilities
- Biometric parameters
- …..



Market consistent valuation e.g. MCEV

IFRS/Local GAAP/Business plan

Fair value balance sheets for:

SST and Solvency II

Properties: Dynamic model

Various interactions/Loops

Stochastic calculation

Complex

- **Accurate** calculation of all embedded options and guarantees
- **Slow**

Core of all modern valuation frameworks: The Baloise case

- Economic scenarios
- Financial data / Assets
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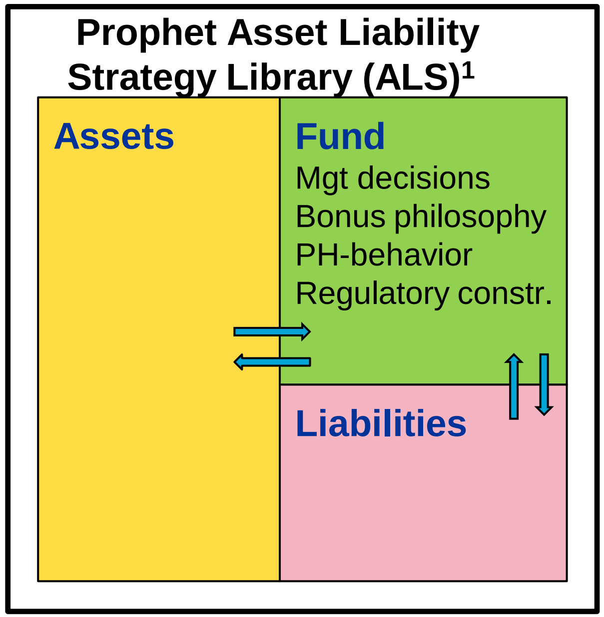
- …..



Market consistent valuation e.g. MCEV

IFRS/Local GAAP/Business plan

Fair value balance sheets for:

SST and Solvency II

Properties: Realistic management decision rules with good backtesting results

ALS with monthly exact "external liabilities[1]"

e.g. individual life: 48 dynamic segments

Very slow: MCEV calculation (5'000 simulations)
- individual life on 70 CPUs: 8 h
- group life on 70 CPUs: 20 minutes

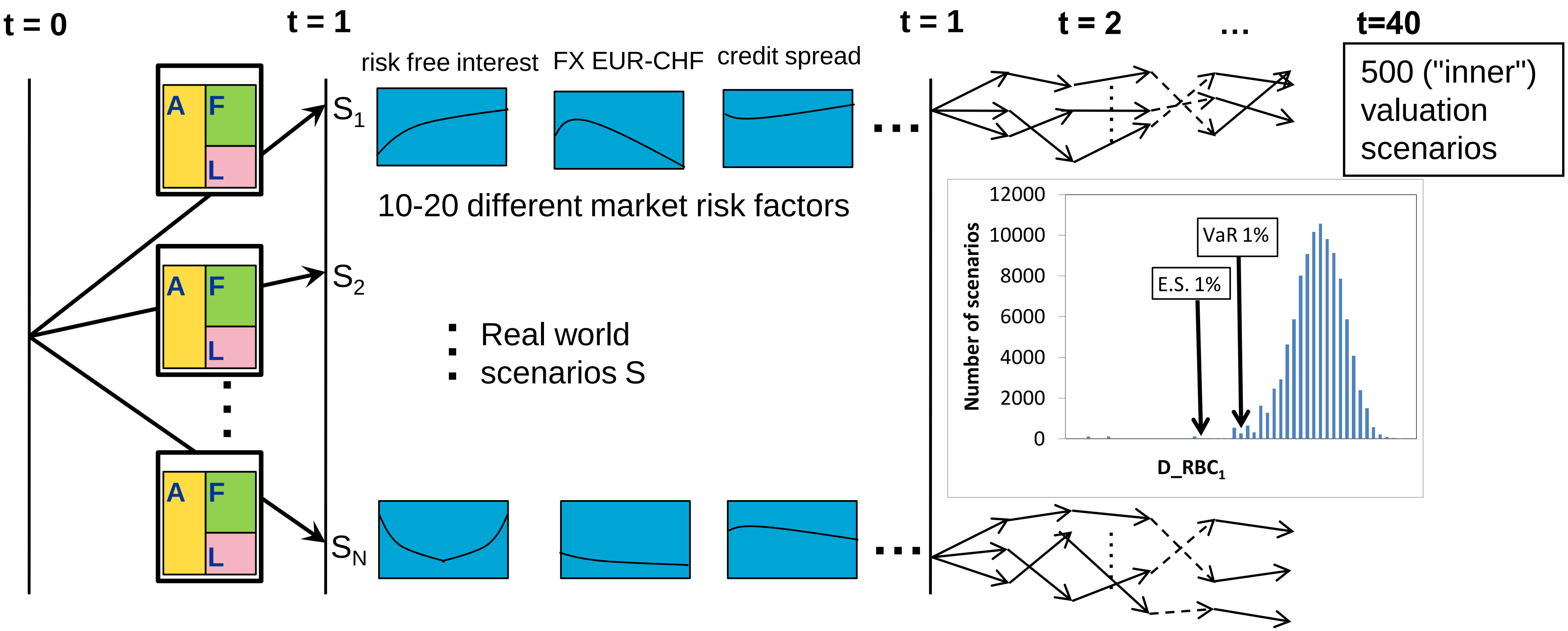
[1] SUNGARD iWorks Prophet

Projection of market consistent balance sheets

Motivation: Assessment of risk exposure requires distribution of risk bearing capital (RBC)

Goal: Accurate calculation of the distribution of the RBC at t = 1

Embedded options and guarantees → Stochastic calculation



Reasonably accurate calculation of expected shortfall (1%) requires about 100'000 ("outer") real world scenarios.

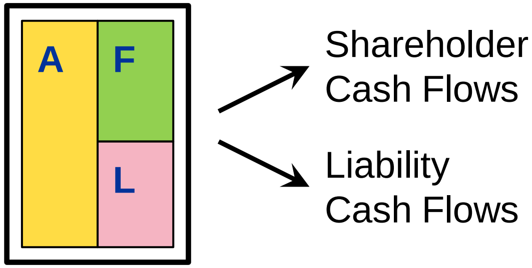
→

Runtime of calculation becomes prohibitive

ind. life: 100'000 x 500 x 8h/5000 = 80'000 h ≈ 475 weeks

group life: 100'000 x 500 x 1/3 h/5000 = 3'333 h ≈ 20 weeks

Common approach: Use of replicating portfolio (RP)



Strategy:

- Ignore complexity of model (e.g. management rules)
- Find replicating portfolio for liability CFs: $\text{RP(L)} = \sum \text{candidate assets}$
- Candidate assets: ZCB, puts, calls, swaptions, …

Requirements: $\text{PresentValue(L)} \cong \text{PresentValue(RP(L))}$

$$\frac{\partial}{\partial \text{risk}_{\text{factor}}} \text{PresentValue(L)} \cong \frac{\partial}{\partial \text{risk}_{\text{factor}}} \text{PresentValue(RP(L))}$$

Candidate assets analytically tractable → Solvency capital calculation straightforward

Challenges:

- Hard to find good RP[2]
- Criteria for goodness-of-fit, metric[2]
- Overfitting: Subspace of null vectors is huge[2]
- Quality of roll forward of RP(L), quality of RP(L) in tail

} Predictive power questionable? Model validation?

- Impossible to replicate cash flows of path dependent options (e.g. rolling means, legal quote, bonus philosophy) with path independent candidate assets

Our conclusion: Establishment of reliability of RP would be a formidable task → Look for an alternative

[2] Presentation J. Crugnola/A. Meister: Stochastic Uncertainties in Market Consistent Valuation

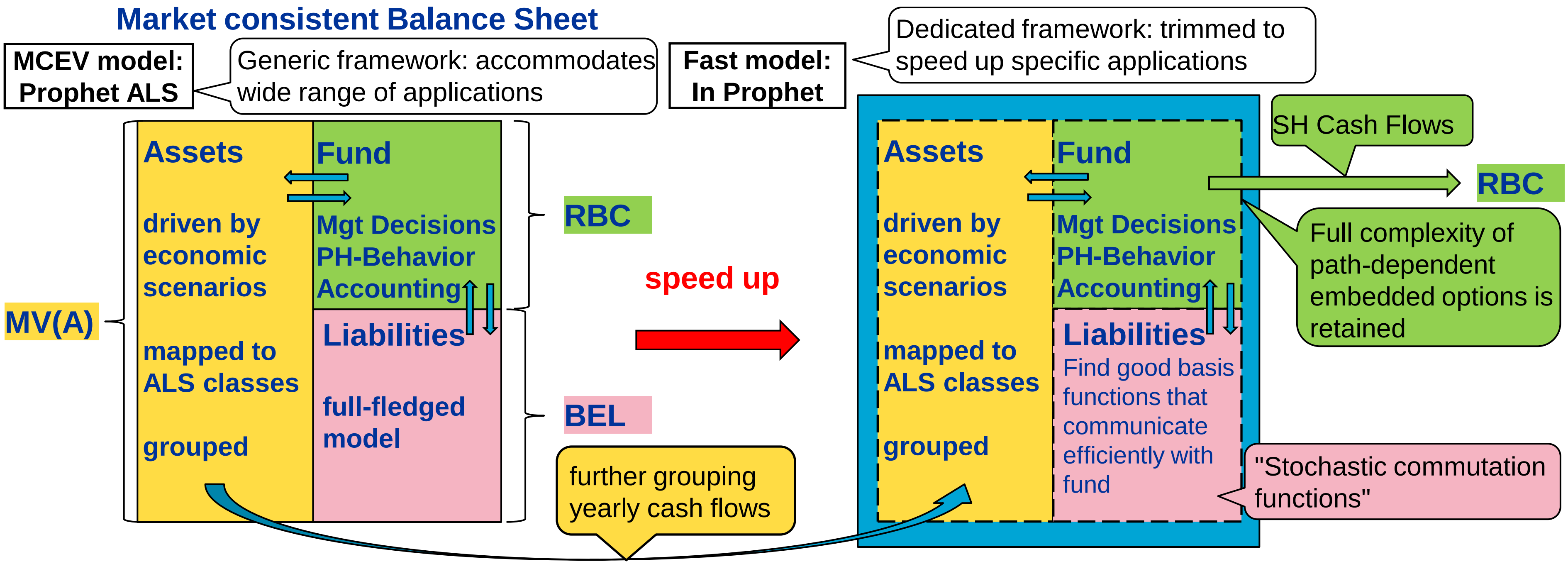
Crucial steps to make nested stochastic simulation feasible

- Try to solve a harder problem!
- Find a ~~RP~~ fast tool that replicates all cash flows in every projection year.
- Solve the right problem! Do not ignore the dynamics of the "fund".
- In a first step it might help to integrate replicating portfolio techniques into the dynamic model[3]
- Then think about the run-time-inefficiencies in your models, look at a TEV-calculation for example…
- Analyse your MCEV model in detail
- In a further step replace in your "vision" the word "proxy" by "perfect analytical / algebraic replication"

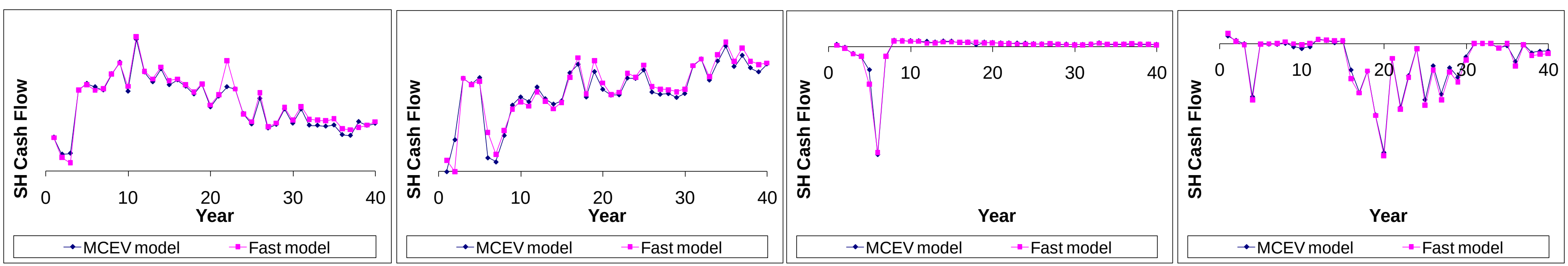
→ This analysis requires a deep understanding of your MCEV model, on the actuarial as well as on the soft- and hardware level.

[3] Case study with SUNGARD

Our approach: Construction of a fast model

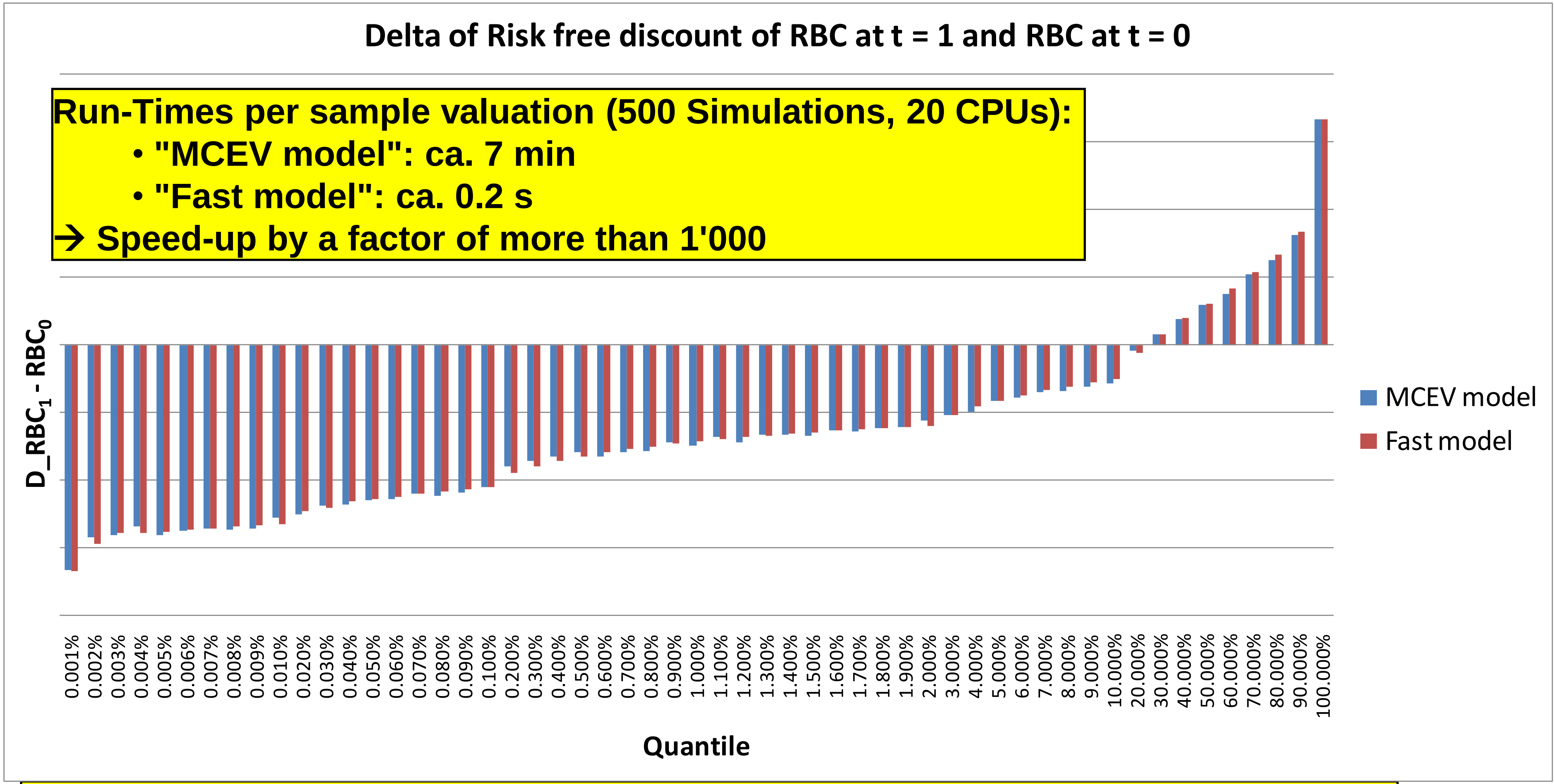


Aim: Replication of cash flows in *every* scenario in *every* year.



Quality test of fast model: Group life

Comparing sample valuations at t = 1 using the "MCEV model" and the "fast model". Sampling over the full range of the distribution of RBC at t = 1:



→The quality is uniform over the whole range of scenarios from worst to best.

Advantages of nested stochastics techniques: Modeling Aspects I

- Conceptually very clean method
- Sound economic interpretation of the results
- 50 Mio. simulations in 12 h instead of 475 weeks (for individual life)
- Calculation of distribution of $RBC_{t=1}$ in 12 h → VaR_{α}, ES_{α} straightforward
- Validation "fast" versus "MCEV model" in 2-3 days
- Calibration, i.e. transformation of "MCEV model" → "fast model" with fast robust algorithm, no "art" or "trial and error"

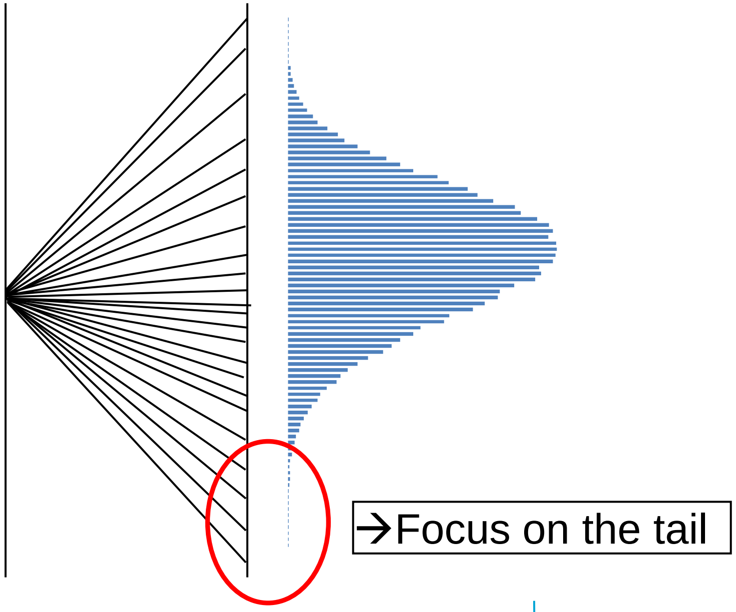
Advantages of nested stochastics techniques: Modeling Aspects II

- Dynamic model calculation without grouping of model points seems to be accessible. Main problem: memory (de)allocation
- "Fast model" completely equivalent to "MCEV model" (e.g. update of management decision report with over 160 sensitivities)
- The group life model allows for an analytical and economically interpretable basis of "candidate liabilities" (summarized in 3 pages of concise formulae)
- Whether 50 Mio. simulations are sufficient or not is an inherent problem of all stochastic techniques. An acceleration of O(1) is always possible by brute force hardware improvements.

Advantages of nested stochastics techniques: ALM aspects

- The "fast models" are complete ALM-tools

  If you change tactic or strategic asset allocation, there is *no need* for a recalibration
- ALM and SST calculations are "consistent"
- Good platform for developing optimal hedging strategies
- Efficient "risk dimension reduction methods" can be implemented

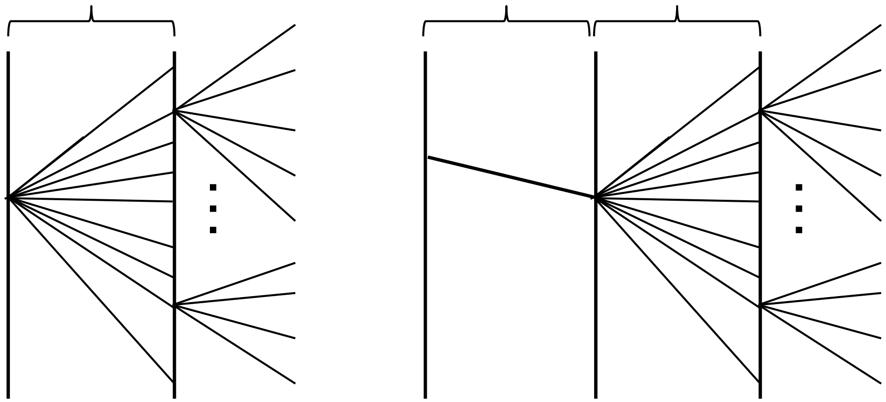


Advantages of nested stochastics techniques: Solvency aspects

- Easy to verify other proxy methods[4]
- Confidence intervals for expected shortfall easily accessible:

  Apply standard statistical methods to the tail of the pertinent distribution, which is conveniently described by generalized Pareto-distribution[5]
- Other model enhancements:

  Replace 1 year by τ 1 year



τ = 1, 2, ..., 12 months: monthly SST

τ = 1, 2, ..., 5 years: ORSA-like solvency reporting

[4] Ambrus et al., *Interest Rate Risk: Dimension Reduction in the Swiss Solvency Test*

[5] Fuhrer, Schmid, *On the accuracy of solvency calculations*, in preparation

Embedding all Solvency calculation methods in one unifying framework

- Initial data (Policy and Asset Data): $m = (m_1, m_2, \ldots, m_p) \in R^p$
- Scenario data: $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_s) \in R^s$
- Contribution of one simulation path σ to the RBC after one year: $RBC_1(\sigma, m)$
- Tensor Product Decomposition[6]: $F(R^s \times R^p) \ni RBC_1(\sigma, m) \cong \sum_j c_j(\sigma) \cdot B_j(m) \in \underbrace{F(R^s)}_{\text{Space of functions of the scenario data}} \otimes \overbrace{F(R^p)}^{\text{Space of functions of the initial data}}$

  (exact equality may require "infinite sums")

In principle, all "proxy techniques" used for Solvency calculations can be written in this way, e.g.:

– SST Standard model ("Delta-Gamma") and other "formula fitting" methods:

First derivatives of the RBC w. r. t. the risk factors ("Delta")

$$RBC_1 = RBC_0 + \sum_l \Delta r_l \cdot B_l^{(1)} + \frac{1}{2} \sum_{k,l} \Delta r_k \Delta r_l \cdot B_{kl}^{(2)} + \ldots$$

Second derivatives of the RBC w. r. t. the risk factors ("Gamma")

– RP: For fixed m, the $c_j(\sigma) = A_j(\sigma)$ are the candidate assets, the $B_j = B_j(m)$ are the coefficients determined by regression:

$$RBC_1(\sigma, m) \cong \sum_j A_j(\sigma) \cdot B_j$$

– Our nested stochastics approach: $c_j(\sigma)$ and $B_j(m)$ are both determined algebraically using algorithms. No fitting of coefficients is involved as in other approaches.

[6] Stone-Weierstrass theorem

Conclusions

- Full nested stochastics is challenging but feasible
- Prerequisites: Understand your model, your software and your hardware
- Once the right problems are identified and solved, the model structure is preserved in full integrity: Powerful and transparent method for ALM/solvency considerations
- Access to the distribution of RBC opens the door to a plethora of new applications
- Nested stochastics models allow one to address the relevant task:
  Providing concise information for the management to drive decisions

References

- Case Study SUNGARD,

  http://www.sungard.com/~/media/financialsystems/casestudies/iworks_casestudy_baloise.ashx
  http://www.prophet-web.com/assets/Datasheets/BaloiseCaseStudy.pdf

- J. Crugnola-Humbert, A. Meister, *Stochastic Uncertainties in Market Consistent Valuation,* SAV General Assembly, August 28th 2009, Luzern

- M. Ambrus, J. Crugnola-Humbert, M. Schmid: "Interest Rate Risk: Dimension Reduction in the Swiss Solvency Test", *EAJ* **1** (2) (2011), 159-172

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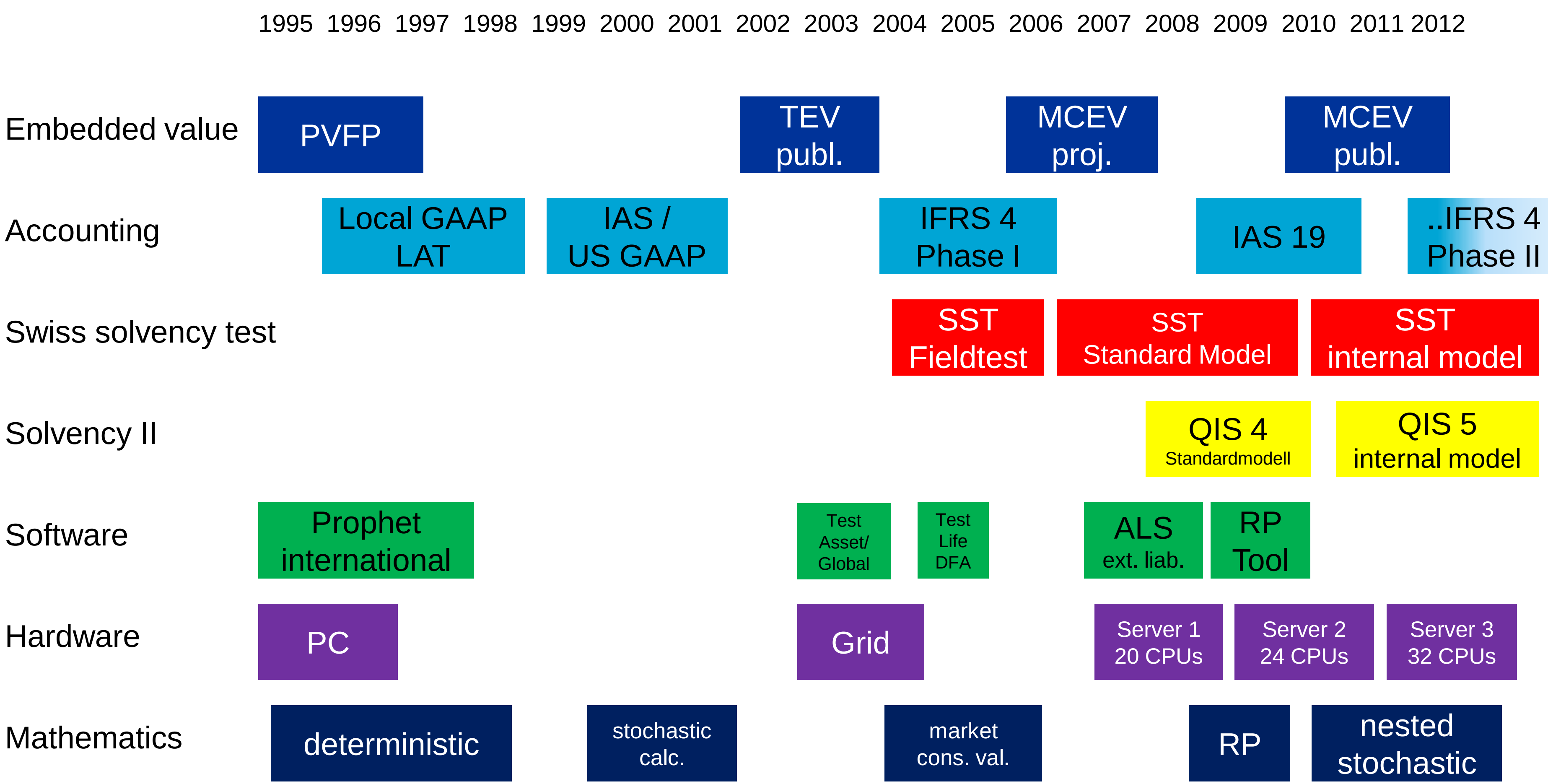
- M.H. Stone, "Applications of the Theory of Boolean Rings to General Topology", TAMS, **41** (3), (1937), 375

Back-up

Evolution of valuation frameworks & software @Baloise.Switzerland



Confidence intervals for expected shortfall:

- Theorem of extreme value theory:

  For a large class of distributions *F* of the variable *X*, the distribution of the excess losses F_u over the threshold *u*,

$$F_u(y) = \mathrm{P}[X - u \le y \mid X > u]$$

  converges to a generalized Pareto distribution[7] $G_{\xi,\beta}$ with parameters ξ and $\beta(u)$.

- The pertinent tail of the distribution of -ΔRBC can accurately be described by a generalized Pareto distribution, given the estimate $F(u) \approx (N - N_u)/N$ with N_u the number of data points with values above threshold *u* in a sample of size *N.* VaR_α and ES_α have the simple form

$$\mathrm{VaR}_\alpha = u + \frac{\beta}{\xi}\left[\left(\frac{N}{N_u}(1-\alpha)\right)^{-\xi} - 1\right], \quad \mathrm{ES}_\alpha = \frac{\mathrm{VaR}_\alpha + \beta - \xi u}{1-\xi}.$$

- Standard statistical methods allow for a straightforward determination of confidence intervals[8] of VaR_α and ES_α for α close to 1 (in our case α ≥ 0.92 roughly).

[7] for details, see
Balkema, A., and de Haan, L. (1974), *Annals of Probability*, **2**, 792–804.
Pickands, J. (1975), *Annals of Statistics*, **3**, 119–131.

[8] for an overview of the method, see e.g.
McNeil A., Frey R., Embrechts P., *Quantitative Risk Management: Concepts, Techniques and Tools,* (2005), Princeton University Press.

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